

Thermal self-energies at zero momentum

José F. Nieves

Department of Physics, University of Puerto Rico
P. O. Box 23343, Rio Piedras, Puerto Rico 00931

Palash B. Pal

Indian Institute of Astrophysics, Bangalore 560034, INDIA

Revised January 1995

LTP-041-UPR, IIA-NAPP-94-14

Abstract

In general the zero momentum limit of thermal self-energies calculated in perturbation theory depends on the order in which the time and the space components of the momentum are taken to zero. We show that this is an artifact of the perturbative calculation, and in fact the non-analyticity of the one-loop self-energy disappears when it is calculated with improved vertices and/or improved propagators that incorporate the imaginary part of the self-energy.

The existing calculations of thermal self-energy functions using the formalism of Thermal Field Theory yield results that are not defined if the external momentum 4-vector is zero. The classic example is the photon self-energy $\pi_{\mu\nu}$ in an electron gas. It is well known [1] that the result of the one-loop calculation of $\pi_{\mu\nu}(k^0, \vec{k})$ for a photon with external momentum $k^\mu = (k^0, \vec{k})$ is such that

$$\lim_{|\vec{k}| \rightarrow 0} \pi_{\mu\nu}(0, \vec{k}) \neq \lim_{k^0 \rightarrow 0} \pi_{\mu\nu}(k^0, \vec{0}), \quad (1)$$

so that the limit in which all components go to zero is not defined. To be more specific, this inequality is obtained only for the real part of the self-

energy. For the imaginary part, the two limits coincide and hence it is well defined.

There have been two opposite attitudes to this result. On one side it has been argued that the inequality should be expected on physical grounds, since the quantity $\lim_{|\vec{k}|\rightarrow 0} \pi_{\mu\nu}(0, \vec{k})$ describes the screening of static electric fields in the long wavelength limit, whereas $\lim_{k^0\rightarrow 0} \pi_{\mu\nu}(k^0, \vec{0})$ gives the plasma oscillation frequencies in the quasi-static limit, and they should be different. Others believe that this is a mere restatement of the inequality in words rather than in symbols, and it does not explain really why the limit of a static homogeneous field should be a pathological one. Indeed, Gribovsky and Holstein [2] even argued, using an effective Lagrangian approach, that the limit of zero momentum is a perfectly well-defined one. We will side with this second group and consider the inequality of Eq. (1) as a problem, or a puzzle.

Various attempts have been made to resolve this puzzle [2, 3, 4, 5, 6], which involve either introducing new and ad-hoc Feynman rules for thermal field theories, or putting restrictions on the general rules [7]. Along another line of attempt, it has recently been pointed out by Arnold, Vokos, Bedaque and Das (AVBD) [8] that the problem mentioned above occurs only if the self energy diagram contains two propagators of the same mass. If the masses of the particles in the loop are different, the problem does not exist. In fact, the calculations of the neutrino self-energy in a gas of electrons and nucleons, which were carried out even before the work of Ref. [8], show this feature explicitly [9, 10, 11]. It has been speculated that this property may be utilized to introduce a mass-splitting regularization for thermal diagrams[8] in cases where problems are known to occur.

This problem, as well the attempts to resolve it, are based on the results of one-loop perturbative calculations. It is natural to ask whether the singular behaviour of the self-energy function at zero momentum might be a consequence of the approximations and idealizations that are implicitly made in the perturbative calculations. In this article we show that this is precisely the case.

We begin our analysis by showing that the problem does not exist in some theories even if the virtual lines in the loop carry particles of the same mass. The example that we give explicitly in this context is the model of a fermion ψ interacting with a pseudoscalar ϕ via a Yukawa-type interaction

of the form $\bar{\psi}\gamma_5\psi\phi$. The 1-loop self-energy of the ϕ particle in this model is well defined in the zero momentum limit. We mention that something similar occurs if the interaction is taken to be a derivative coupling of the form $\bar{\psi}\gamma_\mu\gamma_5\psi\partial^\mu\phi$. While this result is not too surprising since the low momentum limit of this theory and the theory with the pseudoscalar coupling is similar, this example illustrates in a particularly simple way that the zero momentum singularity in some cases disappears if the self-energy diagram is calculated with an improved vertex. We then consider some classes of models in which, as we show, the 1-loop calculation of the self-energy yields a result that is defined at zero momentum when it is carried out using improved propagators for the particles that appear in the internal lines of the loop diagrams. Some of these propagators have an absorptive part which, as observed by AVBD[8], is well-defined at zero momentum even if they are calculated to one-loop and the particles in the internal lines of the loop have the same mass. As we will see, the absorptive part of such diagrams in turn governs the zero-momentum limit of those self-energy diagrams in which the internal lines have the same mass. The conclusion that emerges is that, in theories where the problem exists, the singular nature of the 1-loop self-energies in the zero momentum limit signal a breakdown of the perturbative expansion in that momentum regime. In those cases, the calculation must be carried out using an improved vertex and/or improved propagators in which the absorptive effects are taken into account. In a theory with gauge invariance, such as Scalar Electrodynamics or QED proper, it is necessary to use both the improved vertex and the improved charged-particle propagator since any modification to either one of these quantities must be accompanied by a modification to the other, in order to satisfy the Ward identity and thereby guarantee gauge invariance.

Before proceeding, we recapitulate some results of the canonical approach to the thermal propagators, which we will be using throughout [12]. In this approach, one has to use anti-time-ordered propagators in addition to the time-ordered ones, as well as propagators with no time-ordering. These propagators can be arranged in the form of a 2×2 matrix. For example, for any quantum field Φ^A where A denotes any Lorentz index carried by the

field (none for a scalar field, a Dirac index for the fermion), we can write

$$i\mathbf{D}_{11}^{AB}(x-y) \equiv \langle \mathcal{T} \Phi^A(x) \bar{\Phi}^B(y) \rangle, \quad (2)$$

$$i\mathbf{D}_{22}^{AB}(x-y) \equiv \langle \bar{\mathcal{T}} \Phi^A(x) \bar{\Phi}^B(y) \rangle, \quad (3)$$

$$i\mathbf{D}_{12}^{AB}(x-y) \equiv \mathcal{F} \langle \bar{\Phi}^B(y) \Phi^A(x) \rangle, \quad (4)$$

$$i\mathbf{D}_{21}^{AB}(x-y) \equiv \langle \Phi^A(x) \bar{\Phi}^B(y) \rangle, \quad (5)$$

where the bar denotes complex conjugation for bosons and Dirac conjugation for fermions and \mathcal{F} is defined by

$$\mathcal{F} = \begin{cases} -1 & \text{for fermions,} \\ 1 & \text{for bosons.} \end{cases} \quad (6)$$

\mathcal{T} and $\bar{\mathcal{T}}$ are the time-ordering and anti-time-ordering operators defined as

$$\mathcal{T} \Phi^A(x) \bar{\Phi}^B(y) \equiv \Theta(x_0 - y_0) \Phi^A(x) \bar{\Phi}^B(y) + \mathcal{F} \Theta(y_0 - x_0) \bar{\Phi}^B(y) \Phi^A(x) \quad (7)$$

$$\bar{\mathcal{T}} \Phi^A(x) \bar{\Phi}^B(y) \equiv \Theta(y_0 - x_0) \Phi^A(x) \bar{\Phi}^B(y) + \mathcal{F} \Theta(x_0 - y_0) \bar{\Phi}^B(y) \Phi^A(x) \quad (8)$$

where Θ is the step function. We now write the momentum space expansion of the field in the form

$$\Phi^A(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{\lambda} \left[a_{\lambda}(p) u^A(p, \lambda) e^{-ip \cdot x} + b_{\lambda}^*(p) v^A(p, \lambda) e^{ip \cdot x} \right], \quad (9)$$

where u^A and v^A represent different plane wave solutions arranged by the index λ , and $a_{\lambda}(p)$ and $b_{\lambda}(p)$ are the annihilation operators for particles and antiparticles respectively (for a self-adjoint field $a_{\lambda}(p) = b_{\lambda}(p)$). The properties of the thermal bath come in from the expectation values

$$\langle a_{\lambda}(p) a_{\lambda'}^*(p') \rangle = (2\pi)^3 2E \delta(\vec{p} - \vec{p}') \delta_{\lambda\lambda'} [1 + \mathcal{F} f(p, \alpha)], \quad (10)$$

$$\langle b_{\lambda}(p) b_{\lambda'}^*(p') \rangle = (2\pi)^3 2E \delta(\vec{p} - \vec{p}') \delta_{\lambda\lambda'} [1 + \mathcal{F} f(p, -\alpha)], \quad (11)$$

with

$$f(p, \alpha) = \frac{1}{e^{\beta p \cdot u - \alpha} - \mathcal{F}}, \quad (12)$$

where α plays the role of a chemical potential. We have introduced the velocity 4-vector u^μ of the heat bath, which has components $(1, \vec{0})$ in its own rest frame.

For future purpose, we introduce the matrix \mathbf{U} :

$$\mathbf{U} = \frac{1}{\sqrt{1 + \mathcal{F}\eta(p, \alpha)}} \begin{pmatrix} 1 + \mathcal{F}\eta(p, \alpha) & \Theta(-p \cdot u) + \mathcal{F}\eta(p, \alpha) \\ \Theta(p \cdot u) + \mathcal{F}\eta(p, \alpha) & 1 + \mathcal{F}\eta(p, \alpha) \end{pmatrix}, \quad (13)$$

where

$$\eta(p, \alpha) = \Theta(p \cdot u)f(p, \alpha) + \Theta(-p \cdot u)f(-p, -\alpha), \quad (14)$$

for both bosons and fermions, provided we use the appropriate form for the function f . In cases where specifically the bosonic or the fermionic form has to be used, we will use a subscript B or F to indicate that fact. Thus, the procedure described above gives [12] the 2×2 scalar field thermal propagator as

$$\Delta(p) = \mathbf{U}_B \begin{pmatrix} \Delta_0 & 0 \\ 0 & -\Delta_0^* \end{pmatrix} \mathbf{U}_B, \quad (15)$$

where

$$\Delta_0 \equiv \frac{1}{p^2 - M^2 + i0}, \quad (16)$$

and for fermions

$$\mathbf{S}(p) = \mathbf{U}_F \begin{pmatrix} S_0 & 0 \\ 0 & -\hat{S}_0 \end{pmatrix} \mathbf{U}_F, \quad (17)$$

where

$$\begin{aligned} S_0 &= \frac{1}{\not{p} - m + i0}, \\ \hat{S}_0 &= \gamma_0 S_0^\dagger \gamma_0. \end{aligned} \quad (18)$$

It is easily checked that this gives, for example, the propagators

$$\begin{aligned} \mathbf{S}_{11}(p) &= (\not{p} + m_\psi) \left[\frac{1}{p^2 - m_\psi^2 + i0} + 2\pi i \delta(p^2 - m_\psi^2) \eta_F(p, \alpha_\psi) \right] \\ \Delta_{11}(p) &= \frac{1}{p^2 - M_\phi^2 + i0} - 2\pi i \delta(p^2 - M_\phi^2) \eta_B(p, \alpha_\phi) \end{aligned} \quad (19)$$

for a fermion ψ and scalar ϕ , which are the ones given by Dolan and Jackiw [13]. The explicit forms of the other components are also given in the literature [14]. We now investigate the implications of these Feynman rules on self-energy diagrams in various models.

Model 1

Consider the pseudoscalar interaction

$$L_{\text{int}} = if\bar{\psi}\gamma_5\psi\Phi + \lambda\Phi^4. \quad (20)$$

Notice that the Lagrangian in this case obeys a parity symmetry

$$\Phi \rightarrow -\Phi, \quad \psi \rightarrow \gamma_0\psi, \quad (21)$$

under which these are the only possible renormalizable interaction terms. A cubic Φ^3 interaction, for example, is not invariant under this symmetry. Using the free-field propagator in Eq. (19) for the fermion field we obtain

$$\begin{aligned} \text{Re } \Pi_{11}^{(\Phi)}(k_0, \vec{k}) &= 4f^2 \int \frac{d^4p}{(2\pi)^3} \eta_F(p, \alpha_\psi) \delta(p^2 - m_\psi^2) \\ &\times \left\{ \frac{p \cdot k}{k^2 + 2p \cdot k} + (k \rightarrow -k) \right\}, \end{aligned} \quad (22)$$

which has the unique limit

$$\text{Re } \Pi_{11}^{(\Phi)}(0, \vec{0}) = 4f^2 \int \frac{d^3p}{2E(2\pi)^3} (f_\psi + f_{\bar{\psi}}), \quad (23)$$

where we have denoted by $f_{\psi, \bar{\psi}} = f_F(p, \pm\alpha_\psi)$ the fermion and antifermion momentum distributions. In Eq. (22) the vacuum contribution has been omitted, as we will always do henceforth whenever we write explicit expressions for the self-energies.

In passing, we note that the property of a unique zero-momentum limit does not follow if the fermion-boson interaction is scalar rather than pseudoscalar. On the other hand, if the fermion bilinear is either vector or axial vector type, with a derivative coupling to the spin-0 boson, the result of the 1-loop diagram gives $\text{Re } \Pi_{11}^{(\Phi)}(0, \vec{0}) = 0$ independently of how the limit is

taken. Of course, such couplings can arise only if the theory at hand is an effective one. Nevertheless, it shows the possibility that the zero momentum singularity goes away if the self-energy is calculated with an effective vertex, that may itself be the result of an improved calculation, instead of the fundamental one. In the case of the vector coupling, the parity transformation of Eq. (21) is not a symmetry, so it is likely that the theory includes a trilinear coupling Φ^3 , which gives a contribution to the self-energy that contains the zero momentum singularity.

Model 2

Consider a scalar Φ interacting with another (charged) scalar ϕ with the following interaction Lagrangian

$$L_{\text{int}} = (f\phi\phi\Phi^* + H.c.) + \lambda_\phi|\phi|^4 + \lambda_\Phi|\Phi|^4 \quad (24)$$

It can be easily seen that these are the most general renormalizable interaction terms if the Lagrangian obeys a global U(1) symmetry under which the charges of ϕ and Φ are 1 and 2 respectively. This symmetry automatically rules out any cubic self-interaction of any of the scalar fields. This model is similar to the following one

$$L_{\text{int}} = (f\psi_L^T C\psi_L\Phi^* + H.c.) + \lambda|\Phi|^4 \quad (25)$$

in which Φ interacts with a Weyl fermion.

Now consider, for example, the one-loop diagram for the Φ self-energy in a background of ϕ particles, depicted in Fig. 1. The result of calculating that diagram using the free-field propagator given above for the ϕ field is

$$\begin{aligned} \text{Re } \Pi_{11}^{(\Phi)}(k_0, \vec{k}) &= 2f^2 \int \frac{d^4p}{(2\pi)^3} \eta_B(p, \alpha_\phi) \delta(p^2 - M_\phi^2) \left(\frac{1}{k^2 - 2p \cdot k} \right) \\ &= 2f^2 \int \frac{d^3p}{(2\pi)^3 2E} \left[\frac{f_\phi}{k^2 - 2p \cdot k} + \frac{f_{\vec{\phi}}}{k^2 + 2p \cdot k} \right], \end{aligned} \quad (26)$$

where $p^\mu = (E, \vec{p})$ with $E = \sqrt{\vec{p}^2 + M_\phi^2}$ and we have put $f_{\phi, \vec{\phi}} = f_B(p, \pm\alpha_\phi)$ to denote the momentum distributions. The above formula reveals the problem to which we alluded in Eq. (1). The same behaviour is obtained for the photon self-energy in a background of electrons, or charged scalars. The

main observation of this paper is that, as we already mentioned, this problem vanishes if the diagram is evaluated employing the full propagator of the ϕ field instead of the free-field propagator, which is what we show now.

In what follows we will treat in detail only the two scalar model defined by Eq. (24), since the analysis and results are similar for the other model of Eq. (25). As we mentioned earlier, this problem vanishes if the diagram is evaluated employing the full propagator of the ϕ field instead of the free-field propagator. The full ϕ propagator, which we denote by $\Delta^{(\phi)'}(p)$, can be written just like in Eq. (15) but with Δ_0 replaced by

$$\Delta_0^{(\phi)'} = \frac{1}{p^2 - M_\phi^2 - \Pi_0^{(\phi)}}, \quad (27)$$

where $\Pi_0^{(\phi)}$ is the self-energy function for the ϕ field. Thus,

$$\Delta^{(\phi)'}(p) = U_B \begin{pmatrix} \Delta_0^{(\phi)'} & 0 \\ 0 & -\Delta_0^{(\phi)'} \end{pmatrix} U_B, \quad (28)$$

and in particular,

$$\Delta_{11}^{(\phi)'}(p) = \frac{1}{p^2 - M_\phi^2 - \Pi_0^{(\phi)}} - 2\pi i \rho^{(\phi)}(p) \eta_B(p, \alpha_\phi), \quad (29)$$

with the spectral density $\rho^{(\phi)}$ given by

$$\pi \rho^{(\phi)}(p) = - \frac{\text{Im } \Pi_0^{(\phi)}}{(p^2 - M_\phi^2 - \text{Re } \Pi_0^{(\phi)})^2 + (\text{Im } \Pi_0^{(\phi)})^2}. \quad (30)$$

As the interactions are turned off, $\rho^{(\phi)}$ approaches an on-shell delta function. However, the existence of a non-zero $\text{Im } \Pi_0^{(\phi)}$ smears the delta function that is present in Eq. (19). For this reason it is easy to see that, if Diagram 1 is calculated with the propagator $\Delta_{11}^{(\phi)'}$ for the ϕ field instead of the free particle propagator, then the problem of the non-analyticity of $\Pi_{11}^{(\Phi)}(k)$ at zero momentum disappears. Notice that the dispersive part of $\Pi_0^{(\phi)}$ plays no role in this argument. It is not difficult to see that retaining only the dispersive part of $\Pi_0^{(\phi)}$ and neglecting its absorptive part does not remove the singularity at zero momentum of $\Pi_{11}^{(\Phi)}(k)$.

The next step is to calculate $\Pi_0^{(\phi)}$ and show that in general it has an absorptive part. To this end, we recall that the inverse of the full scalar propagator is given by

$$\Delta'^{-1}(p) = p^2 - M^2 - \mathbf{\Pi}, \quad (31)$$

where $\mathbf{\Pi}$ is a 2×2 matrix whose components must be calculated using the Feynman rules of the theory. Comparing Eqs. (28) and (31), the following relations are obtained [12]:

$$\mathbf{\Pi}_{11} = \Pi_0 + (\Pi_0 - \Pi_0^*)\eta_B(p, \alpha) \quad (32)$$

$$\mathbf{\Pi}_{22} = -\Pi_0^* + (\Pi_0 - \Pi_0^*)\eta_B(p, \alpha) \quad (33)$$

$$\mathbf{\Pi}_{12} = -(\Pi_0 - \Pi_0^*)\epsilon(p \cdot u)f_B(p, \alpha) \quad (34)$$

$$\mathbf{\Pi}_{21} = -(\Pi_0 - \Pi_0^*)\epsilon(-p \cdot u)f_B(-p, -\alpha). \quad (35)$$

From these, it is easily seen that

$$\text{Re } \Pi_0(p) = \text{Re } \mathbf{\Pi}_{11}(p) \quad (36)$$

$$\text{Im } \Pi_0(p) = \frac{\epsilon(p \cdot u)\mathbf{\Pi}_{12}(p)}{2if_B(p, \alpha)}. \quad (37)$$

Therefore, to determine $\Pi_0^{(\phi)}$ we must calculate $\mathbf{\Pi}_{11}^{(\phi)}$ and $\mathbf{\Pi}_{12}^{(\phi)}$, which can be done by evaluating the diagrams in Fig. 2. Since the dispersive part $\Pi_0^{(\phi)}$ is not relevant for resolving the zero momentum problem of the Φ self-energy, we will not calculate it here. However, for the consistency of our scheme it is important to stress that since the internal lines in Fig. 2 correspond to particles of different mass, the function $\text{Re } \mathbf{\Pi}_{11}^{(\phi)}$ (and hence $\text{Re } \Pi_0^{(\phi)}$) does not suffer from the zero momentum problem according to the observation of AVBD [8].

We now turn the attention to the calculation of the absorptive part of $\Pi_0^{(\phi)}$. The simplest way to proceed is to calculate $\mathbf{\Pi}_{12}^{(\phi)}$ and then use Eq. (37). Writing

$$k = p' + p, \quad (38)$$

the application of the Feynman rules to the diagram of Fig. 2 gives

$$-i\mathbf{\Pi}_{12}^{(\phi)}(p) = (if)(-if) \int \frac{d^4 p'}{(2\pi)^4} i\Delta_{21}^{(\phi)}(p') i\Delta_{12}^{(\Phi)}(k), \quad (39)$$

where the scalar propagators are given by

$$\begin{aligned}\Delta_{12}^{(\Phi)}(k) &= -2\pi i \delta(k^2 - M_\Phi^2) f_B(k, \alpha_\Phi) \epsilon(k \cdot u), \\ \Delta_{21}^{(\phi)}(p') &= 2\pi i \delta(p'^2 - M_\phi^2) f_B(-p', -\alpha_\phi) \epsilon(p' \cdot u)\end{aligned}\quad (40)$$

Substituting these propagators into Eq. (39) and using Eq. (37) we obtain

$$\begin{aligned}\text{Im } \Pi_0^{(\phi)}(p) &= (-\pi f^2) \epsilon(p \cdot u) \int \frac{d^4 p'}{(2\pi)^3} \delta(k^2 - M_\Phi^2) \delta(p'^2 - M_\phi^2) \\ &\quad \times \epsilon(p' \cdot u) \epsilon(k \cdot u) (f_B(p', \alpha_\phi) - f_B(k, \alpha_\Phi)).\end{aligned}\quad (41)$$

In writing this formula we have used the identity

$$f_B(-p', -\alpha_\phi) f_B(k, \alpha_\Phi) = f_B(p, \alpha_\phi) [f_B(p', \alpha_\phi) - f_B(k, \alpha_\Phi)], \quad (42)$$

which follows from momentum conservation and the fact that the chemical potentials satisfy

$$\alpha_\Phi = 2\alpha_\phi, \quad (43)$$

as a consequence of charge conservation. Eq. (41) can be written in the form

$$\text{Im } \Pi_0^{(\phi)} = -|p \cdot u| \Gamma(p), \quad (44)$$

where we have defined

$$\begin{aligned}\Gamma(p) &\equiv \frac{f^2}{2p \cdot u} \int \frac{d^3 k}{(2\pi)^3 2\omega} \frac{d^3 p'}{(2\pi)^3 2E'} (2\pi)^4 \\ &\quad \times \{ \delta^{(4)}(p + p' - k) [f_\phi(1 + f_\Phi) - f_\Phi(1 + f_\phi)] \\ &\quad + \delta^{(4)}(p - p' - k) [(1 + f_\phi)(1 + f_\Phi) - f_\phi f_\Phi] \\ &\quad + \delta^{(4)}(p + p' + k) [f_\phi f_{\bar{\Phi}} - (1 + f_\phi)(1 + f_{\bar{\Phi}})] \\ &\quad + \delta^{(4)}(p - p' + k) [f_{\bar{\Phi}}(1 + f_{\bar{\phi}}) - f_{\bar{\phi}}(1 + f_{\bar{\Phi}})] \} .\end{aligned}\quad (45)$$

For the sake of brevity, we have put

$$\begin{aligned}f_{\phi, \bar{\phi}} &= f_B(p', \pm \alpha_\phi), \\ f_{\Phi, \bar{\Phi}} &= f_B(k, \pm \alpha_\Phi),\end{aligned}\quad (46)$$

to denote the particle and antiparticle distributions, and in addition,

$$p'^\mu = (E', \vec{p}'), \quad E' = \sqrt{\vec{p}'^2 + M_\phi^2}, \quad (47)$$

$$k^\mu = (\omega, \vec{k}), \quad \omega = \sqrt{\vec{k}^2 + M_\Phi^2}. \quad (48)$$

The formula for Γ given in Eq. (45) is immediately recognized as the total rate for a ϕ particle of energy p^0 and momentum \vec{p} (as seen from the rest frame of the medium) with integrations over the phase space weighted by the statistical factors appropriate for each process [15]. Notice that f^2 is the amplitude for the decay processes $\phi\bar{\Phi} \rightarrow \bar{\phi}$ and $\phi \rightarrow \bar{\phi}\Phi$, for the annihilation processes $\phi\phi \rightarrow \Phi$ and $\phi\phi\bar{\Phi} \rightarrow 0$, as well as for the inverse reactions of all of them. For certain specific values of p^0 and \vec{p} some of these processes will be kinematically forbidden, but in general Γ is non-zero.

In conclusion, we have presented various models that have similar particle content but different interactions, which exemplify several situations. In some models the one-loop self-energies are well defined at zero momentum while in others the zero momentum singularity disappears if the coupling is taken to be an effective vertex with a suitable momentum dependence. In another type of model, the singularity disappears if the self-energy is calculated with improved propagators that include the absorptive part. Carrying over this idea to the case of QED, which would require that we also use an improved vertex in order to maintain the Ward identity and guarantee a gauge invariant result, it implies that the one-loop photon self-energy calculated with the full charged particle propagator instead of the free propagator is defined at zero momentum. In particular, the absorptive part of the charged particle self-energy, which physically is related to the damping rate of the particle, cannot be neglected if the photon self-energy is evaluated at zero momentum. Then, the physical picture that emerges is the following. The traditional formulas that are given for

$$\lim_{|\vec{k}| \rightarrow 0} \pi_{\mu\nu}(0, \vec{k}) \quad (49)$$

and

$$\lim_{k^0 \rightarrow 0} \pi_{\mu\nu}(k^0, \vec{0}), \quad (50)$$

which are related to well known physical quantities such as the plasma frequency and Debye radius, are valid in the limiting cases

$$k^0 = 0 \quad ; \quad \Gamma \ll |\vec{k}| \ll m \quad (51)$$

$$\vec{k} = 0 \quad ; \quad \Gamma \ll k^0 \ll m, \quad (52)$$

where m stands for the charged particle mass. Since the two limits correspond to two different physical situations the results are different. Traditionally Γ

is omitted in the above conditions, but then it must be kept in mind that the formulas cannot be taken literally all the way to zero momentum.

References

- [1] See, e.g., H. A. Weldon, Phys. Rev. D26, 1394, (1982).
- [2] P. S. Gribosky, B. R. Holstein, Z. Phys. C47, 205 (1990).
- [3] H. Matsumoto, Y. Nakano, H. Umezawa, Phys. Rev. D31, 1495 (1985).
- [4] Y. Fujimoto, H. Yamada, Z. Phys. C37, 265 (1988).
- [5] T. S. Evans, Z. Phys. C41, 333 (1988).
- [6] P. F. Bedaque and A. Das, Phys. Rev. D45, 2906 (1992).
- [7] See H. A. Weldon, Phys. Rev. D47, 594 (1993), for an enlightening discussion of the issues and subtleties involved.
- [8] P. Arnold, S. Vokos, P. Bedaque, A. Das, Phys. Rev. D47, 4698 (1993).
- [9] D. Nötzold and G. Raffelt, Nucl. Phys. B307, 924 (1988).
- [10] P. B. Pal and T. N. Pham, Phys. Rev. D40, 714 (1989).
- [11] J. F. Nieves, Phys. Rev. D40, 866 (1989).
- [12] J. F. Nieves, Phys. Rev. D42, 4123 (1990).
- [13] L. Dolan, R. Jackiw, Phys. Rev. D9, 3320 (1974).
- [14] See, e.g., Ref. [12] and references therein.
- [15] This result provides another example of the kind of relations discussed in H. A. Weldon, Phys. Rev. D28, 2007 (1983).

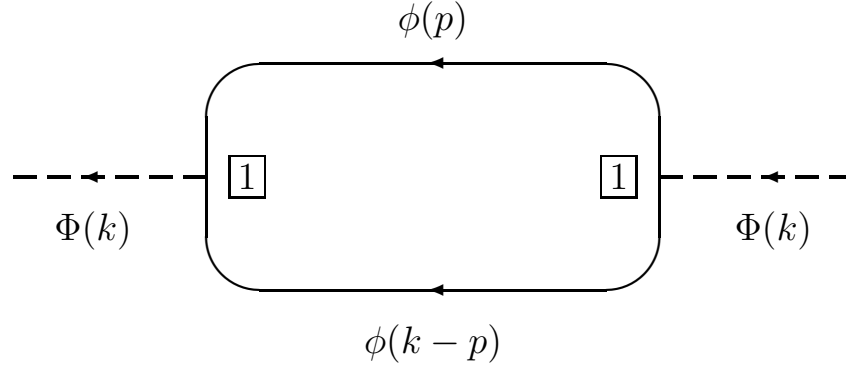


Figure 1: Self-energy diagram for a scalar Φ in a background of ϕ scalar particles in the model of Eq. (24). The type of each vertex used is depicted in a box near the corresponding vertex.

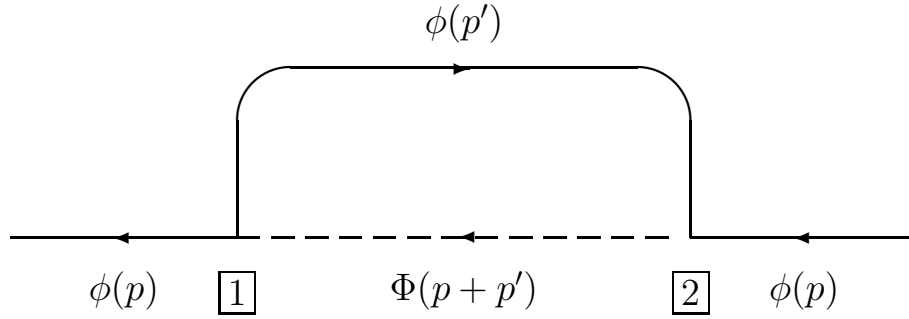


Figure 2: Self-energy diagram for ϕ in the model of Eq. (24). For the calculation of $\Pi_{12}^{(\phi)}$, the left and right vertices should be of type 1 and 2, respectively.